Table 2 Residuals and correlation coefficients

Time, k	Residuals	Time, k	Residuals	Lag, Δt	r_k
1	0.50466	13	3.03277	Δt	0.24663
2	−0.88240	14	−1.24318	$2\Delta t$	0.05611
3	1.83704	15	−0.31517	$3\Delta t$	0.07838
4	−0.03073	16	−0.18640		
5	−0.18433	17	−0.53914		
6	1.06789	18	0.11952		
7	0.87981	19	−0.74797		
8	0.02979	20	0.04551		
9	1.58582	21	−0.83975		
10	−0.02825	22	−0.44318		
11	1.17027	23	0.24139		
12	−2.33596	24	−0.29270		

correlation coefficients at lag k is given by[6]

$$r_k = \frac{C_k}{C_0} \qquad C_k = \frac{1}{N}\sum_{t=1}^{N-k}(z_t - z_{av})(z_{t+k} - z_{av})$$

$$(k = 0, 1, 2, \ldots)$$

and $k<N$, usually of the order of $N/4$, where N is the number of data points, z_t is the random number received at point t, z_{t+k} is the random number received at point $t+k\Delta t$, and z_{av} is the average of all z_t.

If $|r_k|<\gamma$, where γ is some positive constant depending on the accuracy requirements of the user, the Δt that gave us the above r_k is selected. If $|r_k|<\gamma$, then Δt is increased until $|r_k|<\gamma$ is satisfied.

Two simple examples illustrate the previous ideas. We consider the nonlinear system

$$\dot{x}_1 = \tfrac{1}{2}x_2^2 \tag{8}$$

$$\dot{x}_2 = w(t) \tag{9}$$

with $\bar{x}_2(0) = 1$, $\bar{w}(k) = 0$, and $E[w(i)w^T(k)] = 0.2\delta_{jk}$. After linearization about $\bar{x}_2(t) = 1$ and discretization, we find

$$\delta x(k+1) = \begin{bmatrix} 1 & \Delta t \\ 0 & 1 \end{bmatrix} \delta x(k) + \begin{bmatrix} \tfrac{1}{2}\Delta t^2 \\ \Delta t \end{bmatrix} w(k) \tag{10}$$

Let us assume that the measurement model is found as

$$\delta z(k+1) = \delta x_1(k) + v(k+1) \tag{11}$$

where $\bar{v}(k) = 0$ and $E[v(j+1)v^T(k+1)] = \delta_{jk}$.

Finally, it is assumed that tr $P(kk) \leq 1.70$.

The results of these computations with $\Delta t = 1, 2, 5, 10, 20$ s are presented in Table 1. Clearly, $\Delta t = 2$ s satisfies the user's requirements.

The second example concerns the numerical generation of Gaussian white noise with zero mean and variance one, according to one of the standard numberical methods. These numbers are assumed to be the residuals of the system of the previous example where we used $\Delta t = 0.5$ s and $\gamma = 0.06$. The results are shown in Table 2. We notice that for a shift $2\Delta t = 1$ s, $|r_k| < 0.06$ is obtained and, consequently, whiteness is quite good. Finally, if we take $1 \leq \Delta t \leq 2$ s, the best performance of the system is achieved. If $\Delta t > 2$, then the system's performance is below the user's requirements, although the model is valid. If $\Delta t < 1$, the system's model is not valid. The importance of proper selection at Δt then becomes obvious.

Conclusion

The size of the discretization interval Δt effects the performance of hybrid navigation systems. Two bounds for the interval Δt can be specified numerically. Both bounds must be greater than the computation time, otherwise the best Δt is equal to the computation time.

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Concept to Enhance Strapdown Navigation on MaRV's

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Introduction

STRAPDOWN inertial measurement units (IMU's) have long been considered for maneuvering re-entry vehicle (MaRV) navigation because of their potentially lower life-cycle cost, greater reliability, and convenient source of body-frame rate and acceleration data. They are extremely difficult to calibrate, however, and because they must therefore rely on long-term stability of the inertial sensors for performance they have received considerably less attention than their gimballed counterparts for this class of application.

The accuracy is of particular concern for exospherically spin-stabilized MaRV's, where the major source of strapdown navigation error during re-entry is the roll angle error accumulated prior to re-entry. The dominant error source is the roll gyro's scale factor uncertainty. Several projects have been initiated or proposed by government and industry to reduce the uncertainty or mitigate its effect, for example: enhanced long-term stability of gyros (particularly laser gyros), addition of a roll-isolation gimbal, and incorporation of roll-update sensors such as star trackers, gravity gradiometers, magnetometers, horizon scanners, etc.

An alternate concept is presented which essentially requires only a procedural modification at the system level in eliminating the MaRV accuracy sensitivity to the roll angle error accumulated prior to re-entry. Following the boost phase, it is suggested that a MaRV containing a strapdown navigation system be oriented, deployed, and spin-stabilized along the effective or resultant re-entry "drag vector" direction rather than along the conventional re-entry velocity vector direction. For typical maneuvers, the "drag vector" is 10-20 deg away from the re-entry velocity vector. Since the CEP contribution due to roll error is a function of the cross product between the "drag vector" and the spin vector, then forcing them to be colinear results in (theoretically) zero position error.

Received June 25, 1979.

Index categories: LV/M Guidance; Guidance and Controls.

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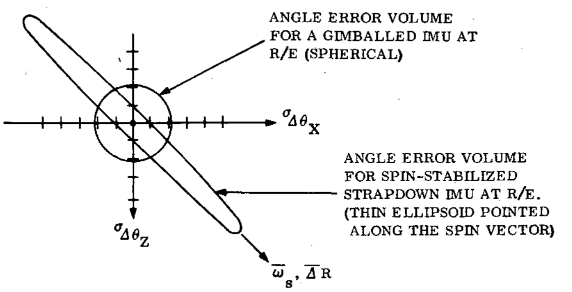

Fig. 1 Comparison of the angle-error characteristics of gimballed and strapdown IMU's.

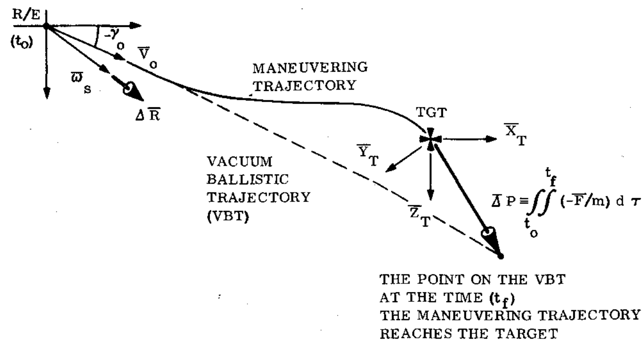

Fig. 2 Illustration of the "drag vector" through which angle errors yield position errors.

Background

In order to clarify the concept proposed, the unique characteristics of the strapdown IMU angle error at re-entry and how it propagates into a position error through re-entry are briefly reviewed.

Angle Error at Re-entry

A strapdown inertial system has to effectively measure the total roll angle traversed by the vehicle during its exospheric spin motion. If the vehicle spins at 2 rps for 25 min, the angle to be tracked is 1,080,000 deg. It the gyro scale factor uncertainty is one part per million (excellent quality), then the roll angle uncertainty at re-entry is 1.08 deg, a large error by inertial standards. Conversely, the spinning motion is beneficial in limiting the angle error accumulated about the inertial directions normal to the spin axis, since the spin has a modulating effect on the bias drift terms of the pitch and yaw gyros. Because of the large roll angle error ΔR and small angle error normal to roll, relative to conventional inertial systems, the shape of the misalignment angle error volume at re-entry is unique for a spin-stabilized strapdown IMU as indicated two-dimensionally in Fig. 1.

The elongated shape should lead one to consider how the system could be used to minimize its effect a la the "diver" concept, i.e., the concept of terminally diving along a preferred line, defined by the system's ellipsoidal position error volume at re-entry, in order to minimize CEP. The suggested approach for minimizing the effect of the roll angle error is fully analogous to the "diver" concept; i.e., in each case the error volume is not reduced, rather the system is made to work such as to minimize its impact on CEP.

Roll Error Propagation into Position Error

In a strapdown navigation system the angle information is used to transform sensed body-frame accelerations into the inertial reference frame. Thus, it is through the specific force vector that angle errors propagate into acceleration errors. The position error sensitivity to angle error is thus the second integral of the acceleration vector. When the vehicle has arrived at the target, the doubly integrated acceleration vector is simply the vector displacement between the target and the vacuum ballistic trajectory at the time of actual vehicle arrival at the target. This displacement vector, $\overline{\Delta P}$ in Fig. 2, is commonly referred to as the "drag vector."

The navigation system's final position error due to the initial roll error $\overline{\Delta R}$ is simply the cross product of $\overline{\Delta R}$ and $\overline{\Delta P}$:

$$\Delta X_T = (\Delta\bar{R} \times \Delta\bar{P}) \cdot \bar{i}_T \tag{1a}$$

$$\Delta Y_T = (\Delta\bar{R} \times \Delta\bar{P}) \cdot \bar{j}_T \tag{1b}$$

$$\Delta Z_T = (\Delta\bar{R} \times \Delta\bar{P}) \cdot \bar{k}_T \tag{1c}$$

Desensitizing the Roll Error

Equations (1) indicate that the sensitivity of position error at the target to roll error at re-entry can be theoretically eliminated by forcing the $\Delta\bar{R} \times \Delta\bar{P}$ cross product to be zero; that is, force $\Delta\bar{R}$ and $\Delta\bar{P}$ to be colinear. This can be achieved most practically by orienting the exospheric spin vector $\bar{W}_S$ along the drag vector $\Delta\bar{P}$. A review of six realistic MaRV trajectories indicates that the $\Delta\bar{P}$ vector is offset from the re-entry velocity vector typically between 10 and 15 deg with a minimum of 4.5 deg and a maximum of 21.3 deg. Offsetting the MaRV spin vector does impose an initial angle of attack which must be trimmed out. Aerodynamic damping on current systems has been shown to be adequate to satisfy this requirement.

The concept was verified using a strapdown error analysis program (SINSEAP) for five MaRV trajectories. In each case, when the spin offset angle was introduced, the position error contributed by roll gyro and computational error sources was reduced to a negligible amount and the total error induced by the MaRV IMU was reduced by as much as 12 to 1.

Concluding Remarks

It has been suggested that spin-stabilized MaRV's using a strapdown IMU be deployed and spun along a direction offset from the normal deployment attitude. It has the potential effect of substantially reducing the accuracy sensitivity of roll gyro scale factor uncertainty. It also has the effect of reducing the sensitivity of the roll gyro bias, roll gyro nonorthogonality, and roll computational error sources as well. It should enable considerable relaxation in the long-term stability requirements of the gyro coefficients without requiring an update sensor or a roll-isolation gimbal. It requires the computation by the fire control system of the MaRV drag vector, which should be readily available from the MaRV targetting data. The orientation has to be computed in any case, it is simply a matter of replacing one computation for another. It does not impact the actual deployment or spin-up. In summary, it is believed that implementation of this concept will enhance the eventual use of strapdown navigation on MaRV's.